

The Role of Problems to Enhance Pedagogical Practices in the Singapore Mathematics Classroom.

Foong Pui Yee

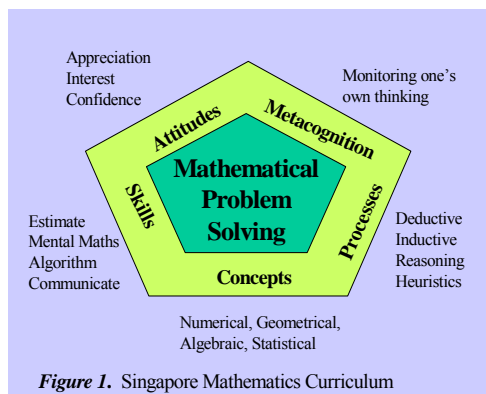
National Institute of Education, Nanyang Technological University, Singapore

Abstract: *The worldwide trend for reform in mathematics education is also happening in Singapore. Teachers are encouraged to use a variety of non-routine mathematics problems in their teaching to de-emphasize the use of routine questions that promote basic algorithmic skills. This paper presents the role of different types of mathematical problems and in particular short, open-ended questions that teachers can modify from the routine questions commonly found in textbooks. The experience of some primary teachers who have used short, open-ended problems with their pupils is shared.*

Introduction

The mathematics curriculum in Singapore is aligned with 21st century reform-based visions of schooling around the world, such as the NCTM standards (NCTM 2000). The primary curriculum aim is to meet the demands of a new century for all students to acquire an understanding of concepts, proficiency with thinking skills, and a positive attitude in knowing that mathematics will be needed for complex and common applications in problem situations. Figure 1 depicts the framework of the Singapore Curriculum (Ministry of Education, 2000) that was conceptualized during the early 90's to encompass *mathematical problem solving* as its core. Mathematical problem solving as stated in this framework includes using and applying mathematics in practical tasks, in real life problems and within mathematics. Through the framework, it is advocated that problems should cover a wide range of situations from routine mathematical problems to problems in unfamiliar context and open-ended investigations that make use of the relevant mathematics and thinking processes.

From the framework in Figure 1, one can recognise that problems are both a means and an end in school mathematics instruction. Teaching via problems serves as a mean for students to construct mathematical concepts and to develop skills. Problems lead students to use heuristics such as investigate and explore patterns and as well as think critically. To solve problems the students must observe, relate, question, reason and infer. Success in problem solving relates to students' disposition and monitoring of their own thinking processes. Mathematics educators, including schoolteachers, are now beginning to pay attention to the kind of tasks they give to students. Currently, most mathematics classrooms in Singapore engage



pupils in the traditional approach of whole class expository teaching followed by pupil practice of routine exercises and regular written tests consisting of multiple-choice questions, short-answer and long-answer, open-response questions (Chang, Kaur, Koay, & Lee, 2001). There is thus a need to equip teachers with a bank of a greater variety of mathematical questions for problem solving that can enhance their teaching options. Pupils must encounter intriguing mathematical problems where they can reason and offer evidence for their thinking, communicate and present their ideas in mathematics, and find connections across mathematics as well as in real life.

The Role of Mathematical Tasks in Thinking and Understanding

Generally mathematics lessons are planned and described in terms of student tasks. These tasks range from simple drill-and-practice exercises to complex problems set in rich contexts for teachers to evaluate students' understanding of the lesson objectives. Yet, according to Carpenter and Lehrer (1999), understanding is not an all-or-none phenomenon. It is a generative process where, when students acquire knowledge with some understanding, they can apply that knowledge to learn new topics and solve new and unfamiliar problems. For understanding to develop, thinking in terms of mental activity must take place. And for thinking to occur, students must be confronted with rich meaningful problems. Carpenter and Lehrer proposed that classrooms need to provide students with tasks that can activate the following five forms of interrelated mental activity for understanding to emerge: (a) develop appropriate relationships, (b) extend and apply their mathematical knowledge, (c) reflect about their own experiences, (d) articulate what they know, and (e) make mathematical knowledge their own. The teaching methods for these thinking activities would include active participation of students, a variety of strategies, attention for interaction and the social aspect of learning.

Teachers in the Singapore primary schools are very conversant with the standard type of problem sums that are set for pupils to work individually, normally after teaching a topic, and equipping them with the necessary basic concepts and procedures to apply. Very often these tasks are termed *one-step, two-step or multiple-step problems* where pupils are taught certain procedures such as model drawing especially for the more complex questions that require understanding of part-whole or proportional relationships. These questions would be formulated in familiar, closed-structured situations such that they became routine exercises after much practice. In recent years, with the emerging trend towards emphasis on *creative and critical thinking skills* in mathematics, teachers have been encouraged to teach problem solving heuristics using non-routine problems that would require strategies such as *guess and check, look for patterns, make supposition, work backwards, and so on*. A recommended list of such heuristics and thinking skills is included in numerous documents including in the MOE syllabus (MOE, 2000). There are abundant resource books available for teaching these skills using non-routine problems, ranging from the classic literature (Polya, 1973; Krulik & Rudnick, 1993) to more recent local adaptations such as Koay (1999) and Yeap (2001). The difficulty confronted by teachers in using such non-routine problems is that these kind of problems are often not content domain-specific and the question of how to integrate them into the *normal* topical mathematics lessons arises. Very often these non-routine mathematical problems are used in supplementary lessons as enrichment for the better ability pupils where the emphasis is on the practice of heuristics with similar kinds of structure. The end results may not be the desired outcome of developing thinking and understanding as advocated by Carpenter and Lehrer (1999). It would instead result in teaching clever tricks to solve certain kind of problems, assuming pupils can recognize them from the onset.

Classification Scheme for Mathematical Problems

In order for teachers to realise the encompassing role of problem solving in the curriculum, they need to distinguish among the various types of problems and their roles. Equipped with knowledge and understanding, they can judiciously select or even construct tasks for their pupils that will promote different forms of thinking activities in mathematics lessons. Based on a systematic search of literature on problem solving and use of problems in research (Foong, 1990), in this paper a classification scheme is proposed for different types of problems that are being encouraged for the 21st century mathematics classrooms, as shown in Figure 2.

We shall adopt the commonly accepted definition of a “problem” as one where thinking takes place when a person is confronted with a problem that has no immediate solution and that the problem solver accepts the challenge to tackle it. This definition excludes those school textbook exercises that are used for practice of an algorithm or skill such as in computational sums or in simple one or two-step

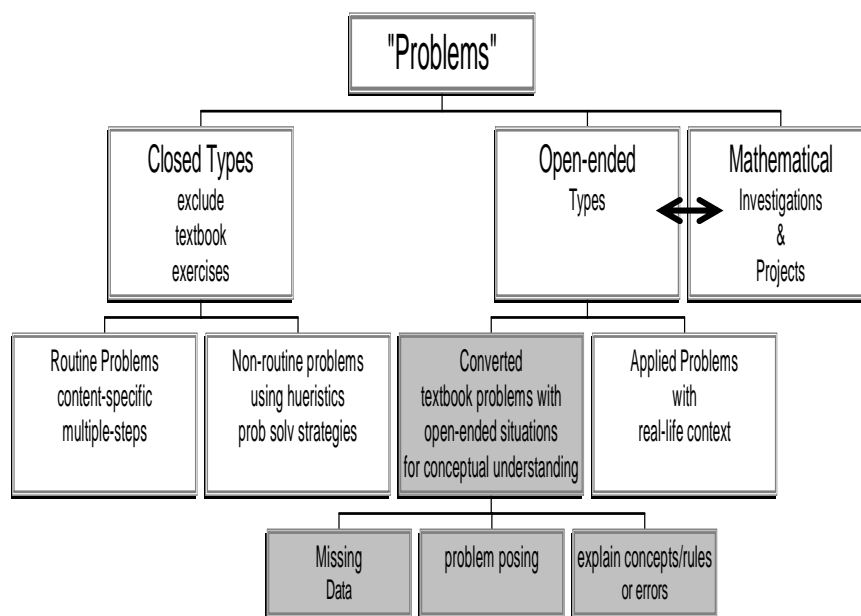


Figure 2. Classification Scheme for Mathematical “Problems”

translation word problems. In this scheme, basically, most *problems* can be broadly classified as “closed” or “open-ended” in structure. Problems in this classification scheme have their different roles in mathematics instruction as in teaching *for* problem solving, teaching *about* problem solving, or teaching *via* problem solving. These roles will be illustrated in the following discussion on the different categories of problems.

Closed Problems

Closed problems are “well-structured” in terms of clearly formulated tasks where the one correct answer can always be determined in fixed ways from the necessary data given in the problem situation. These closed problems would include content-specific routine, multiple-step problems as well as non-routine, heuristic-based problems. To tackle these problems, the solver through productive thinking rather than simple recall must generate some process skills or some crucial steps in the solution method. Figure 3a shows examples of the routine, content-specific, multiple-step problems, also known as “challenge problems” to local teachers, for the topic on fractions. Teachers use such challenge problems for its role in *teaching for problem solving* where the emphasis is on learning mathematics for the main

purpose of applying it to solve problems after learning a particular topic. There is however an over reliance on such type of challenge problems in the Singapore syllabus to assess so called higher-order, analytical thinking skills of the pupils. This also has led to an undesirable practice of over-emphasizing a particular method called *model drawing* taught to children to solve similar structured word problems across related arithmetic topics like whole numbers, fractions, ratio and percent.

Examples of Teaching For Problem Solving:
Problem Sums on Fractions

1. **Minah had a bag of rice. Her family ate an equal amount of rice each day. After 3 days, she had $\frac{1}{3}$ of the rice left. After another 7 days, she had 24 kg of rice left. How much rice was in the bag at first?**
2. **$\frac{3}{5}$ of Pr 6A and $\frac{3}{4}$ of P 6B are girls. Both classes have the same number of girls and P 6A has 8 more boys than P 6B. How many pupils are there in P 6A?**


Figure 3a. Closed content-specific challenge problems

Nevertheless, with the current call for more emphasis on processes, there are recommendations in the syllabus for teachers to use non-routine problems to teach thinking skills and problem solving heuristics. Non-routine, closed problems of the type shown in Figures 3b and 3c are beginning to appear in the mathematics classroom where teachers use them in the role of *teaching about problem solving*. The emphasis is on using heuristic strategies to approach and solve unfamiliar problems that are usually not domain-specific to any topics in the syllabus.

Open-ended Problems

Teaching About Problem Solving:
How many different strategies to solve this problem?

- Farmer Ting was counting his ducks and pigs. He counted 10 heads and 26 feet altogether. How many ducks and pigs does he have?



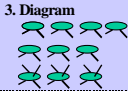
1. Algebraic
 $D+P=10$
 $2D + 4P= 26$

2. Guess and Check
Try 5Ducks, 5Pigs
 $10 + 20$ too much
 $6D, 4P = 28$, too much
 $7D, 3P = 14 + 12 = 26$ OK

OR Tabulate trials

ducks	pigs	heads	total legs
5 x 2	5 x 4	10	30
6 x 2	4 x 4	10	28

3. Diagram




4. Logical reasoning
if all ducks, 20 feet, 6 extras,
3 pairs to pigs,
so 3 pigs, 7 ducks

Figure 3b. Different strategies

Non-routine problem: How many squares are there in a chess board?

Hints: Use the following heuristics

- Work systematically
- try simpler examples
- tabulate results
- look for a pattern
- generalise to a rule



how many squares here
Can you find 5?

Size of Square	Number of squares
2 x 2	5
3 x 3	?
4 x 4	?
.....	
8 x 8	?
n x n	?

Figure 3c. Chessboard problem

This category of problem is often considered as “ill-structured ” for they lack clear formulation - they are missing data or assumptions and there is no fixed procedure that guarantees a correct solution. Many real-world problems, as in Figure 4, would fall into this category. Solving problems from everyday situations requires the individual to begin with a real world situation and then to look for the relevant underlying mathematical ideas.


Practical Real Life Problems: Examples

- **Design a Better Timetable**

Design a better timetable.
Write out your timetable.
Give reasons for your choice to convince others in your group.

 - How long should a period be?
 - Should every week be the same?
 - Assemblies
 - Breaks
 - Which subjects in the afternoon?
 - How long should a day?
 - How much time for each subject?
 - Free periods
 - Groupwork / tutor period
 - etc etc
- **Car Park**

How many cars could be parked on your school playground?
Design a car park layout for your playground?



Consider:
the turning circle of the car
safety for pedestrians
room to open door
different types of vehicle
one-way systems; traffic flow
exits; signs

Figure 4. Examples of Applied Problems with real-life context

Mathematical investigation in the form of major open-ended projects that explore and extend a piece of pure mathematics for its own sake or those real-world problems that require mathematical modeling meant for higher level mathematics will not be discussed in this paper. Such open-ended projects usually require students to demonstrate their ability in the form of detailed reports on how they carry out extended pieces of independent work showing their creative application of mathematical knowledge and skills. This kind of task is used in the formal assessment of students' mathematical power in countries such as Australia (Money & Stephens, 1993) that have adopted a reform-based curriculum.

Converting Textbook Exercises into Short Open-ended Problems

In this paper, we will focus on the types of short, open-ended problems that teachers can convert from closed questions found in textbook exercises. Such tasks can be incorporated into normal lessons and would not require several periods or weeks for pupils to do. Teachers can use short, open-ended problems for their role in *teaching via problem solving* that emphasizes learning mathematical concepts and skills through a problem situation. Carroll (1999) found that short, open-ended questions provided teachers with quick checks into students' thinking and conceptual understanding. They are no more time-consuming to correct than the worksheet questions that teachers normally give. When used regularly, the pupils in the study developed the skills of reasoning and communication in words, diagrams, or picture. In the study, Carroll converted a standard closed question on comparing fraction into

an open-ended situation for pupils to decide and justify their responses; see Figure 5. The responses he received from his pupils to the open-ended questions gave him a deeper insight into the pupils' real understanding of fractions than if he had given them the closed question; see samples in Figure 6. For instance, Warren's and Anna's responses, shown in Figure 6, would have been scored as correct in the closed question. Their drawings show that both would need remedial assistance on the concept of unit fractions and how to represent them. Anna's drawing shows that only she has the correct understanding of the meaning of fraction.

<p>Closed question: Circle the fraction that is bigger</p> <p style="text-align: center;">$\frac{1}{2}$ $\frac{1}{3}$</p>	<p>Open-ended question: If you want the bigger piece of pizza, would you take $\frac{1}{2}$ or $\frac{1}{3}$? <i>Explain your reasoning using word and pictures</i></p>
--	---

Figure 5. Two kinds of questions on comparing fractions

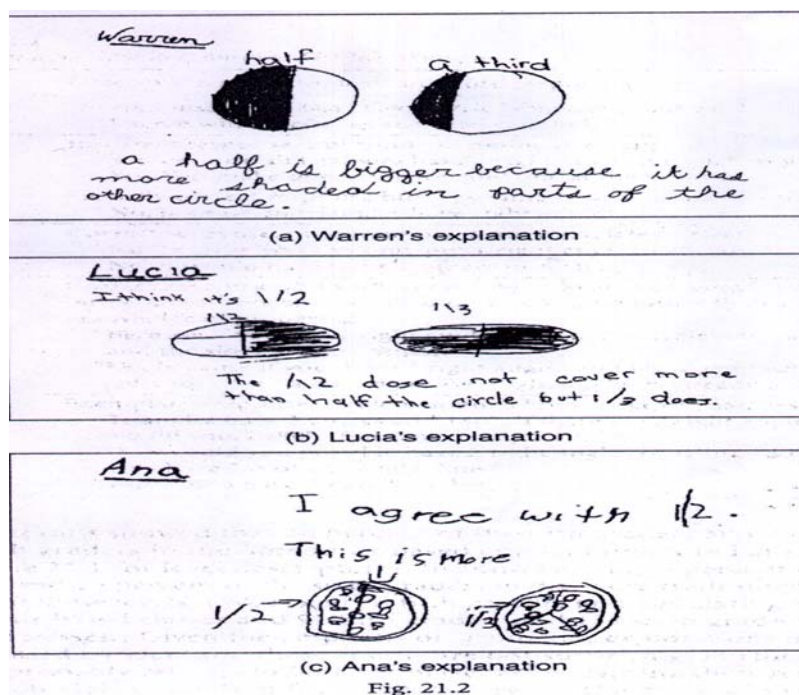


Figure 6. Pupils' responses to an open-ended question (Caroll, 1999)

Consider another typical question that one may find in a primary textbook; see Figure 7. Formulated in such a closed structure, the teacher and pupils would have in mind the expected standard response of seeing it as a multiplication sum. However, in the same context of a Polar bear, teachers in the Netherlands (Van den Heuvel-Panhuizen, 1996) posed an open-ended situation for the pupils to solve, see Figure 8.


Closed Question

- A Polar bear weighs about 20 times as heavy as Ali. If Ali weighs 25 kg. What is the mass of the Polar bear?
- Expected Pupils' responses:
- Cue word: "20 times as heavy"
- Concept: "multiplication" situation
- Procedure: $25 \times 20 = \dots$

$$\begin{array}{r} 25 \\ \times 20 \\ \hline \end{array}$$

Figure 7: Closed question with an expected standard response.

A Polar bear weighs 500 kg. How many children do you need to have the same mass?



Expected Pupils' Responses:

- No fixed cue: can be "division", "multiplication" or "repeated addition", or "ratio"
- Arouses natural curiosity- a real meaningful problem
- Not all data are given
- Pupils to make own assumptions about missing data: weight of a child
- Making decision and estimation on the average weight of a child in relation to themselves

Figure 8: Open-ended situation for a variety of responses.

According to Van den Heuvel-Panhuizen, the Polar Bear problem (Figure 8) represents an important goal of mathematics education. In an open-ended situation, pupils, in addition to applying calculation procedures, are also required to solve realistic problems where there is no known solution beforehand and not all data is given. It would require pupils' own contributions, such as making assumptions on the missing data. Without giving the children's weight, it becomes a real problem and the pupils have to think about and estimate the weight of an average child. There is no cue word for students to figure out which operation to use as in the closed question, Figure 7. When this question was given to the Grade 3 children in the Netherlands, they could use in various ways the mathematics concepts and operations they had learned previously. The open-ended situation enabled the pupils to show what they knew; it also enabled the teachers to acquire richer information on how their pupils tackled problems. The children used their acquired knowledge of measurement, varied 'division' strategies, and different models and notation schemes to support their thought processes. From this study, it shows that short, open-ended tasks used in a constructivist approach can activate understanding and thinking in pupils. Pupils will be given opportunity to articulate what they know when they have to explain the *how* and *why* as they develop appropriate relationships across ideas, connect and apply their mathematical knowledge and make mathematical knowledge their own.

Singapore Primary Teachers' Workshop and Pupils' Work

Since 1999, several local teachers attended workshops conducted by the author on the use and construction of open-ended problems in the teaching of primary mathematics. These workshops constituted one of the many units in their in-service training programme. In the workshop, teachers were exposed to the different kinds of short, open-ended questions; they were given opportunity to convert textbook questions into open-ended problems; and then had to trial their questions on their own pupils in school. The features and types of short, open-ended questions that they can construct are shown in Figures 9 and 10.

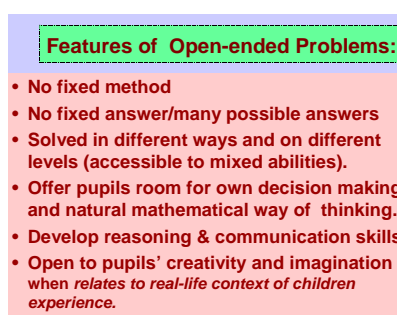


Figure 9. Problem Features

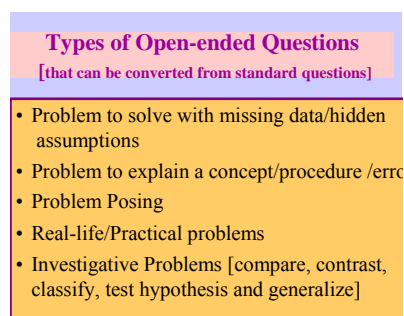


Figure 10. Problem Types

Samples of Teachers' Open-ended Questions and Pupils' Work

The teachers found little difficulty in constructing the questions but initially many of them were apprehensive about giving their pupils such tasks. Such questions had never been part of their teaching where pupils were required to give explanations and reasons for their solutions. The practice has always been that pupils are given problems with only one answer and one taught method of finding it. When the teachers had to trial the open-ended questions with their pupils as part of their assignments in the workshop, many of them were surprised by the various and rich responses that most of their pupils could give. Of course, there were also reports by some teachers that many pupils seem to lack the reasoning and communication skills that they hoped to see in the pupils' work. Some of the teachers gave the problems to their pupils to work in small groups to generate discussion among them. The following are some samples of the teachers' questions and their pupils' responses:

Problem with missing data

Miss A, a primary two teacher formulated this open-ended question as an antithesis to the routine exercises that her pupils were doing on simple word problems in addition and subtraction:

There are some apples on the table and some apples in a small basket. If there are 50 apples altogether, how many apples are on the table. Explain your answer.

She decided to have the pupils worked individually as she felt they were too young and were not used to working in groups for mathematics. To her surprise, most of her pupils were able to respond appropriately and the variety of the children's thinking was enlightening. Some children could reflect upon analysing the question, indicating that it cannot be solved as there was not enough given information. There was a child who was able to generalize that there were many possible answers. Different strategies were used: subtraction, division, and even model drawing. The uses of verbal, symbolic and pictorial mathematical representations were manifested in the children's work when they were given the opportunity to be creative in an open-ended situation such as this. Four pieces of the primary two children's work are shown in Appendix A.

Problem Posing

Mrs. B teaches a primary five class and had completed the unit on data collection and bar graphs. Instead of giving her pupils an extra worksheet of exercises on reading and interpreting information from given graphs, she decided to give small group work on an open-ended problem where pupils have to make up a story from a blank graph; see Figure 11. One of the groups created a meaningful story, in Figure 12, that was based on a real-life event that happened recently in school about a toilet survey. The correctness of the use of graph and data representation, as well as the communication skills (albeit, spelling error of *principal*) in their story, shows that given the opportunity pupils can exercise their critical thinking skills to make connections between mathematics and the real world.

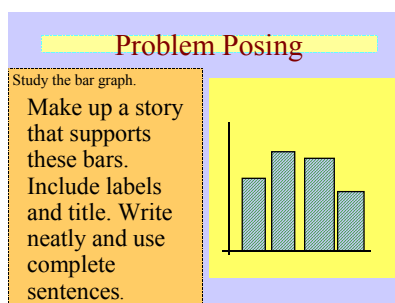


Figure 11. Problem Posing

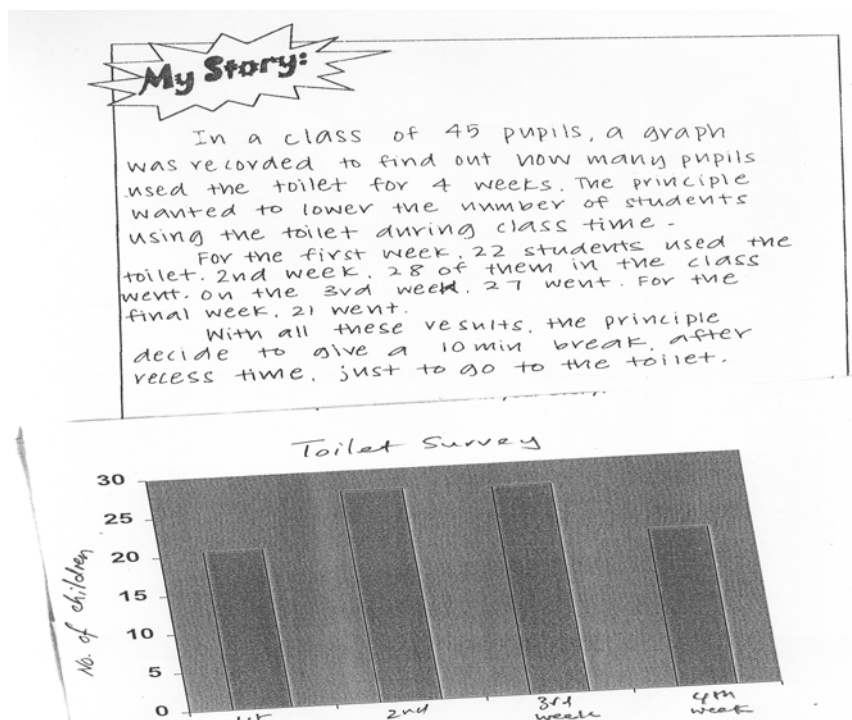


Figure 12. Pupils' story on toilet survey

Another teacher Miss C, formulated this open-ended problem posing, Figure 13, for her primary five girls to assess their understanding on the concept, *average*. She analysed their work and classified them. From the pupils' work she realized that although many of her pupils were able to apply the procedure for finding average in their standard workbook, there were some who had no understanding of the concept based the questions that these pupils had constructed; see Figure 14. Some had the misconception of average as *the total* or as *in sharing or division*. Interestingly, she also found some pupils had very poor measurement sense in the context they used for their constructed questions; see Figure 15. They would use unrealistic measurement units like *93 cm for a toddler*, *81 kg for an obese hamster*, or *a family using only 93 litres of water in a week*. This might also indicate that pupils could be modeling after some textbook or teacher questions that often do not necessarily relate to the real world. It shows that often pupils do not apply their common-sense knowledge and make realistic consideration the context in a problem. Another insight to some of her female primary 5 girls was their

imaginative or otherwise interest outside of school, as in Figure 16, when they use context such as *boyfriends*, *bald man* and *underwear* in their creations.

Problem Posing

Jang An got all his word problems correct. His workings are shown below. Can you think of word problems that match his workings?

Jang An's Working:
 $81 + 93 + 78 = 252$
 $252 \div 3 = 84$
 Ans. The average is 84

Figure 13

Primary Five Pupils' Creation:
Lack full understanding of the concept of average

- There are 3 classes in an art school. The first class has 81 pupils, the second class has 93 pupils, the third class has 78 pupils. Find the total average of pupils in the art school.
- On Monday, Jang An bought 81 marbles. His mother bought him 93 more marbles and his neighbour gave him 78 more marbles. What is the average number of marbles he got that day.
- He has 81 apples, 93 durians and 78 rambutans. He shared it among his friends. What is the average amount of fruits each of them got?

Figure 14

Pupils' Creation: Lacking Measurement Sense

- Three toddlers are 81 cm, 93 cm and 78 cm tall. What is their average height?
- There were 3 obese hamsters. Hamster A weighed 81 kg, hamster B weighed 93 kg, and hamster C weighed 78 kg. What was their average weight?
- The Quek family used 93 litres of water in the first week of August. They used 78 litres in the 2nd week and 81 litres in the 3rd week. Find the average amount of water they used in a week.

Figure 15

P5 Pupils' Creation: Imaginative & Frivolous

- Esther Lee moved to a different estate 3 times. In the first estate, she made 81 boyfriends, in the second estate she made 93 boyfriends. In the third estate, she made 78 boyfriends. On the average, how many boyfriends did she make in each estate?
- There were 3 bald men. Bald man A had 81 hairs, bald man B had 93 hairs and bald man C had 78 hairs. What was their average amount of hairs?
- There was a robbery one day and three pairs of underwear got robbed. Underwear A was size 81, underwear B was size 93 and underwear C was size 78. What was their average size?

Figure 16

Problem to Explain Meanings of Concepts

After teaching several units in the scheme of work on equivalence and comparison of fractions, Miss D decided to give an open-ended question to her primary four class to assess their conceptual understanding. In order to engage her pupils in small group discussion, the context of the problem was set for these pupils to “teach” someone about comparing fractions. She was pleasantly surprised by the variety of strategies that her pupils had used in explaining the fraction concepts. Shown in Appendix B are two of the different approaches that the pupils had written. Although at primary four, the pupils were taught the analytical method of changing unlike fractions to equivalent fractions for comparison, she found that two groups had used successful and more concrete ways of explaining the concepts with fraction models and real-life examples of sharing cakes. Very revealing is that the

group, which used the analytical abstract method, displayed their partial understanding of the procedure, especially in this instance where the LCM of the denominators became too large and cumbersome to compute.

Conclusion

Although there are several other interesting pupils' responses that could not be accommodated in this paper, from the samples that we have presented, the teachers who have used such short, open-ended questions in their classrooms found many advantages that enhanced their professionalism. It is an approach that enabled them to teach mathematics more in line with the national vision of "Thinking schools, learning nation" where the focus is on thinking and reasoning and characterized by a shift from practicing isolated skills towards developing rich network of conceptual understanding. This approach has enabled teachers to see pupil's thinking rather than the teacher's own thinking through closed questions that have predetermined methods and answers. In the traditional approach, there has been a tendency for pupils to see mathematics as merely practicing one-step, two-step or many-step procedures to find answers to routine problems. On the other hand, the open-ended questions if given on a regular basis would instill in pupils that understanding and explanation are crucial aspects of mathematics. From the evidence presented in the pupils' work, they were capable of communication in mathematics using words, diagrams, pictures or manipulatives. While working in groups or individually, these pupils built on their prior knowledge and reasoning skills. Pupils in presenting their solutions to others could compare and examine each other's method and discoveries from thence they could modify and further develop their own ideas in creative ways.

For concluding remarks, to encourage others to implement open-ended questions in their teaching, excerpts of three of the teachers' observation and reflection after they had trialed their problems with their own pupils, are expressed in the following:

Teacher X: *"A lesson such as an open-ended problem solving was indeed a fruitful one for my pupils and me. There was another avenue to motivate pupils to learn mathematics.When the pupils got started to work in pairs, I could see that they were thinking very hard as to how the work could be done. I heard comments like 'How can a person be so heavy?' and 'It is not possible to have so many passengers in a bus?'. I was glad that they tried to explain and justify to their peers..." (primary four)*

Teacher Y: *"From my pupils' work, I feel that critical thinking can be taught. These pupils were frightened at first to take the first step, for fear of getting the answers wrong. But after seeing their friends' answers*

and reasons, these fears were lifted.....Lastly, the pupils do include their own personal experiences and knowledge. I was quite impressed." (primary four)

Teacher Z: "....Another advantage of open-ended investigation is that learning is more active. Pupils had a great time making wild guesses, trying to defend their procedure. However, our pupils are still not accustomed to accepting other pupils' alternative answers, it will take them some time to recognize the fact that there are sometimes more than one solution to a problem..... Open-ended investigations here no doubt require more preparation and time on the part of the teachers, but judging from the positive learning outcome of the activity carried out from most of the pupils, the pupils will begin to see mathematics can be meaningful and interesting. (primary six)

Note: The author would like to thank all the teachers who participated in the open-ended workshop and especially those teachers whose feedback and pupils' work are shown in this paper.

References

- Chang, S. C. , Kaur, B., Koay, P. L., & Lee, N. H. (2001). An exploratory analysis of current pedagogical practices in primary mathematics classroom. *The NIE Researcher*, 1(2),7-8.
- Carpenter, T. P., & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema & T.A. Romberg (Eds.), *Mathematics classrooms that promote understanding* (pp.19-33). Mahwah, NJ: LEA Publishers.
- Carroll, W. M. (1999). Using short questions to develop and assess reasoning. In L. V. Stiff & F. R. Curcio (Eds.), *Developing mathematical reasoning in grades K-12, 1999 Yearbook* (pp. 247-255). Reston, VA.: NCTM.
- Foong, P. Y. (1990). *A metacognitive-heuristic approach to mathematical problem solving*. Unpublished doctoral dissertation. Monash University. Australia.
- Koay, P. L. (1999). *Mathematics activities for upper primary pupils*. Singapore: Oxford University Press.
- Krulik, S., & Rudnick, J. A. (1993). *Reasoning and problem solving: A handbook for elementary school teachers*. Needham Heights, MA.: Allyn and Bacon, Inc.
- Ministry of Education. (2000). *Revised syllabus for primary mathematics*. Curriculum Planning and Development Division. Ministry of Education. Singapore.
- Money, R., & Stephens, M. (1993). A meaningful grading scheme for assessing extended tasks. In N. L. Webb & A. F. Coxford (Eds.), *Assessment in the mathematics classroom, 1993 Yearbook* (pp.177-186). Reston, VA.: NCTM.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

Polya, G. (1973). *How to solve it*. 2nd Ed. Princeton, NJ: Princeton University Press.

Yeap, B. H. (2001). *Problem solving heuristics for PSLE mathematics*. Singapore: Bookland Publisher.

Van den Heuvel-Panhuizen, M. (1996). *Assessment and realistic mathematics education*. Utrecht: CD-B Press/Freudenthal Institute, Utrecht University.

Author:

Foong Pui Yee, Associate Professor, National Institute of Education, Nanyang Technological University, Singapore. pyfoong@nie.edu.sg

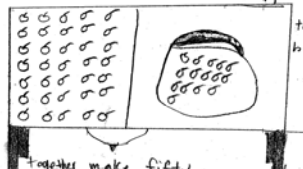
Appendix A

Park View Primary School
P2 Mathematics
HOT Task Question

Name: Angie Lee
Class: P2.2
Date: 2001.4.2

There are some apples on the table and some apples in a small basket.
If there are 50 apples altogether, how many apples are on the table?
Explain your answer.

I think all the answer can make the question
like you see thirty-five and fifteen can make fifty
so if there are thirty-five apples on the table
fifteen apples can be in the small basket.
thirty-five apple · fifteen apple in
the basket.
together make fifty.

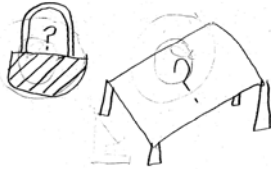


Park View Primary School
P2 Mathematics
HOT Task Question

Name: Isa Razif
Class: P2.2
Date: 24.4.2001

There are some apples on the table and some apples in a small basket.
If there are 50 apples altogether, how many apples are on the table?
Explain your answer.

We don't know the answer
because they did not
tell how many apples is
in the small basket.



...Appendix A

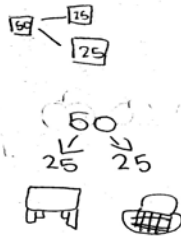
Park View Primary School
P2 Mathematics
HOT Task Question

Name: Wong Dorcas
Class: P2 A
Date: 24/4/2001

There are some apples on the table and some apples in a small basket.
If there are 50 apples altogether, how many apples are on the table?
Explain your answer.

$$50 \div 2 = 25$$

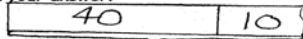
It was because there are two items,
That's why!



Park View Primary School
P2 Mathematics
HOT Task Question

Name: Wun Sook Yee
Class: P2 A
Date: 25-4-2001

There are some apples on the table and some apples in a small basket.
If there are 50 apples altogether, how many apples are on the table?
Explain your answer.



$$40 + 10 = 50$$

There are 40 apple on the table.

It's because the table is bigger than the small basket, so the basket can take less apples

$$\begin{array}{r} 40 \\ + 10 \\ \hline 50 \end{array}$$



